16.8 Videos Guide

16.8a

Theorem (statement):

- Stokes' Theorem: $\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$, where C is the positively oriented piecewise-smooth boundary curve of S, an oriented piecewise-smooth surface
- Green's Theorem as a special case of Stokes' Theorem via a vector form of Green's Theorem
$$\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S} = \iint_D (\operatorname{curl} \mathbf{F}) \cdot \mathbf{k}\, dA$$
- Note that $d\mathbf{S} = \mathbf{n} dS = (\mathbf{r}_u \times \mathbf{r}_v)\, dA$, where $\mathbf{n}$ is a unit normal vector and $\mathbf{r}_u \times \mathbf{r}_v$ is simply a normal vector to the surface S

Exercises:

16.8b

- Verify that Stokes' Theorem is true for the given vector field $\mathbf{F}$ and surface S.
$\mathbf{F}(x, y, z) = -2yz\, \mathbf{i} + y\, \mathbf{j} + 3x\, \mathbf{k}$,
S is the part of the paraboloid $z = 5 - x^2 - y^2$ that lies above the plane $z = 1$, oriented upward

16.8c

- Use Stokes' Theorem to evaluate $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$.
$\mathbf{F}(x, y, z) = x^2 \sin z\ \mathbf{i} + y^2\, \mathbf{j} + xy\, \mathbf{k}$,
S is the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the xy-plane, oriented upward

16.8d

- Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$. In each case C is oriented counterclockwise as viewed from above.
  - $\mathbf{F}(x, y, z) = \mathbf{i} + (x + yz)\, \mathbf{j} + \left(xy - \sqrt{z}\right) \mathbf{k}$,
  C is the boundary of the part of the plane $3x + 2y + z = 1$ in the first octant

16.8e

  - $\mathbf{F}(x, y, z) = 2y\, \mathbf{i} + xz\, \mathbf{j} + (x + y)\, \mathbf{k}$,
  C is the curve of intersection of the plane $z = y + 2$ and the cylinder $x^2 + y^2 = 1$